Lecture – 2
Appendix A: Sections A3, A4
Chapter 2: Sections 1 - 3

Outline

- **Signed Binary Numbers and its operation**
- **Binary Logic and Gates**
- **Circuit Basics**

SIGNED (Positive and Negative) BINARY NUMBERS

How do we present signed numbers: Positive and Negative?

Signed-**Magnitude** Representation

Consists of a magnitude and symbol (sign bit)
The MSB (Most Significant Bit) is the sign bit

- 0 : positive
- 1 : negative

Example: For 8-bit signed magnitude representations:

+ 0001 1010 = 0001 1010
- 0001 1010 = 1001 1010

Whether a number is considered signed or unsigned number is determined by applications. It is most common way of representing the significand in floating point values.

Addition with Signed-magnitude numbers

When signs are same:

- Add and keep the same sign
- Example: 0000 1101 + 0000 0110 = 0001 0011

$(13 + 6 = 19)_{10}$

When signs are not same:

1. Compare the *magnitudes*
2. Subtract/add the smaller magnitude number from/to the larger
3. Give the result the sign of the larger magnitude number
4. **Example**: 1000 1101 + 0000 0110 = 1000 0111

$(-13 + 6 = -7)_{10}$

Signed-Magnitude representation of numbers increases logic-circuit complexity

How to make it simpler?

Other Ways to Represent Signed (Negative) Numbers

Signed-1's-complement

Complement (flip) each bit, including the sign bit,

- that is: 0 -> 1, 1 -> 0

Example:

For 8-bit numbers:

\+ 0001 1010 = 0001 1010
\- 0001 1010 = 1110 0101

Signed-2's-complement

Complement (flip) each bit, including the sign bit and add 1.

- that is: 0 -> 1, 1 -> 0 and then add 1

Example: For 8-bit numbers:

+ 0001 1010 = 0001 1010
- 0001 1010 = 1110 0110

Intuition on why we use 2's complement

Another Way of computing 2's complement	Example 1	Example 2
1. Starting from the right, find the first '1'	0101001	0101100
2. Invert all of the bits to the left of that one	1010111	1010100

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Addition with 2's Complement Numbers

- Positive + positive ~~ same as with signed magnitude numbers

Example: 0000 1101 + 0000 0110 = 0001 0011

$(13 + 6 = 19)_{10}$

- Negative + negative ~~ 2's complement + 2's complement

Example: 1111 0011 + 1111 1010 = 1110 1101

$(-13 - 6 = -19)_{10}$

- Positive + negative ~~ positive + 2's complement

Example:

```
Carries:     1111
             0000  1101                  +13
           + 1111  1010                  -6
           ---------------------------------------------------
             0000  0111                  (13 - 6 = 7)10
```

Subtraction with 2's complement

Subtraction is signed addition

- A - B ~~ A + (-B)
- 6 - 13 ~~ 6 + (-13)
- 6 - (-13) ~~ 6 + 13
- -6 - 13 ~~ -6 + (-13)
- -6 - (-13) ~~ -6 + 13

Examples:

```
Carries  :  1111
            0001   0101              +21
          + 1111   1010               -6
           ------------------------------------------------

           0000   0111               (13 - 6 = 7)10
```

Computers and Electricity

Circuits Gates combined to perform more complicated tasks. It is a hardware component that manipulates binary information. Each basic circuit is logic gate

There are three different, but equally powerful, notational methods for describing the behavior of gates and circuits

1. Boolean expressions
2. Logic diagrams
3. Truth tables

Gate : A device that performs a basic operation on electrical signals. Gates implement logic functions. Performs a specific logical operation.

Boolean Expressions : Expressions in Boolean algebra, is a mathematical notation for expressing two- valued logic. They specify the operation of each digital circuit or gate. It is also used to design logical circuits through the manipulation of Boolean expressions

Logic Diagram : A graphical representation of a circuit - Each type of gate is represented by a specific **graphical** symbol

Truth Table : A table showing all possible input value and the associated output values

Binary Variables : Variables that takes on one of two values.

Binary Variables

- Recall that the two binary values have different names:
  - True/False
  - On/Off
  - Yes/No
  - 1/0

- We use 1 and 0 to denote the two values.

- Variable identifier examples:

  - A, B, y, z, or X_1 for now
  - RESET, START_IT, or ADD1 later

Logical Operations

- The three basic logical operations are:

  - AND
  - OR
  - NOT

- AND is denoted by a dot (·).
- OR is denoted by a plus (+).
- NOT is denoted by a single quote mark (') or an overbar (¯).

Notation Examples

- **Examples:**
  - **Y = A . B** **is read "Y is equal to A AND B."**
  - **Z = x + y** **is read "z is equal to x OR y."**
  - **X = A'** **is read "X is equal to NOT A."**

Truth Tables

- ***Truth table*** **- a tabular listing of the values of a function for all possible combinations of values on its arguments**

Gates

Let's examine the processing of the following six types of gates

1. NOT
2. AND
3. OR

Typically, logic diagrams are black and white, and the gates are distinguished only by their shape

NOT Gate

A NOT gate accepts one input value and produces one output value

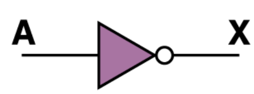
By definition, if the input value for a NOT gate is 0, the output value is 1, and if the input value is 1, the output is 0

A NOT gate is sometimes referred to as an ***inverter*** because it inverts the input value

Boolean Expression

$X = A'$

Logic Diagram Symbol



Truth Table

A	X
0	1
1	0

AND Gate

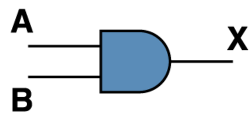
An AND gate accepts two input signals.

If the two input values for an AND gate are both 1, the output is 1; otherwise, the output is 0

Boolean Expression

$$X = A \cdot B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

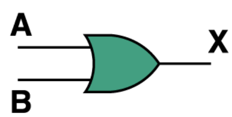
OR Gate

If the two input values are **both** 0, the output value is 0; otherwise, the output is 1

Boolean Expression

$$X = A + B$$

Logic Diagram Symbol



Truth Table

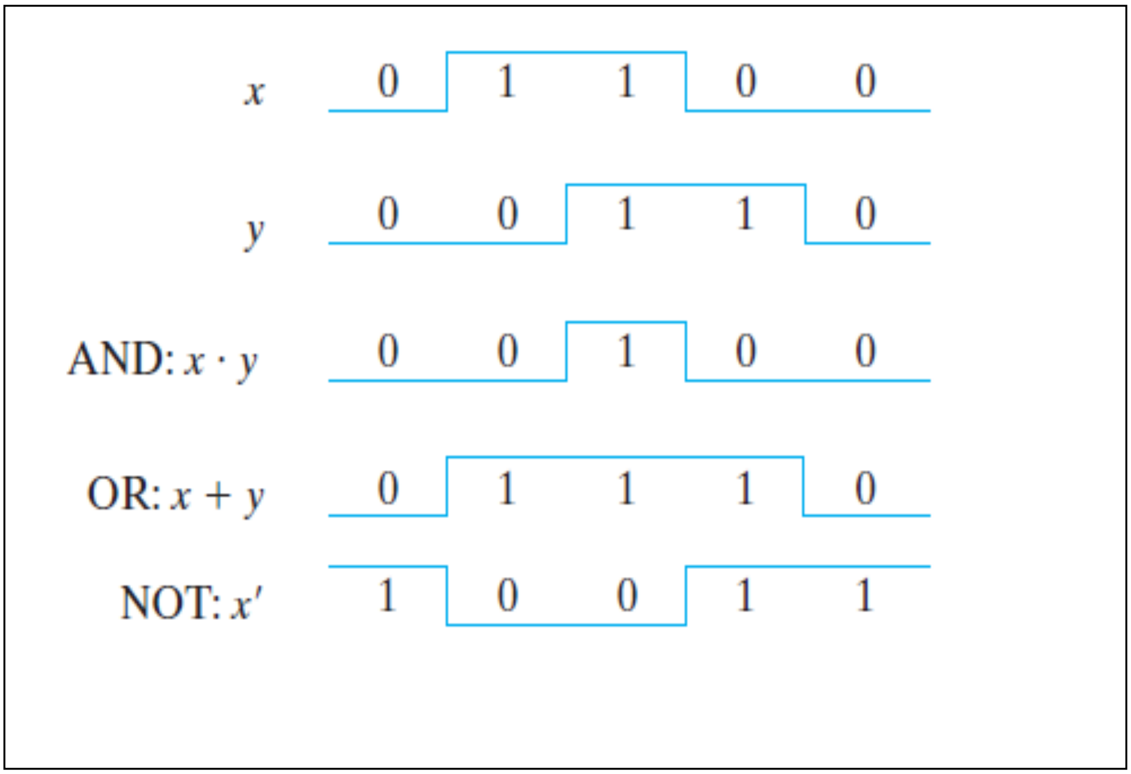
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Review of Gate Processing

- A **NOT** gate **inverts** its single input value
- An **AND** gate produces 1 if both input values are 1
- An **OR** gate produces 1 if one or the other or both input values are 1

Logic Gate Behavior

Waveform behavior in time as follows:



Constructing Gates

Transistor A device that acts, depending on the voltage level of an input signal, either as a wire that conducts electricity or as a resistor that blocks the flow of electricity

- A transistor has no moving parts, yet acts like a switch
- It is made of a semiconductor material, which is neither a particularly good conductor of electricity, such as copper, nor a particularly good insulator, such as rubber

Circuits

Two general categories :

- In a **combinational circuit**, the input values explicitly determine the output
- In a **sequential circuit**, the output is a function of the input values as well as the existing state of the circuit

As with gates, we can describe the operations of entire circuits using three notations

- Boolean expressions
- logic diagrams
- truth tables

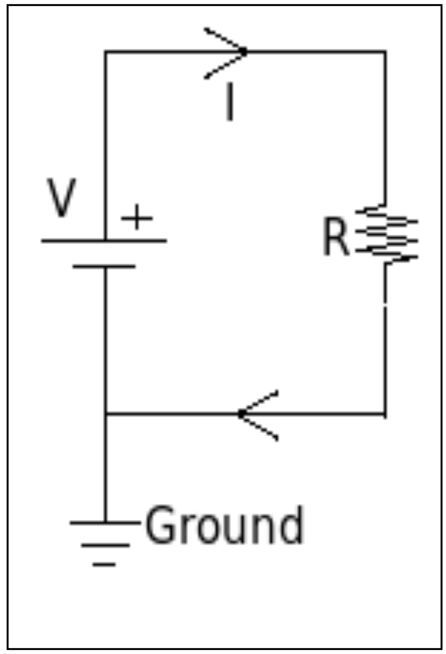
Integrated Circuits

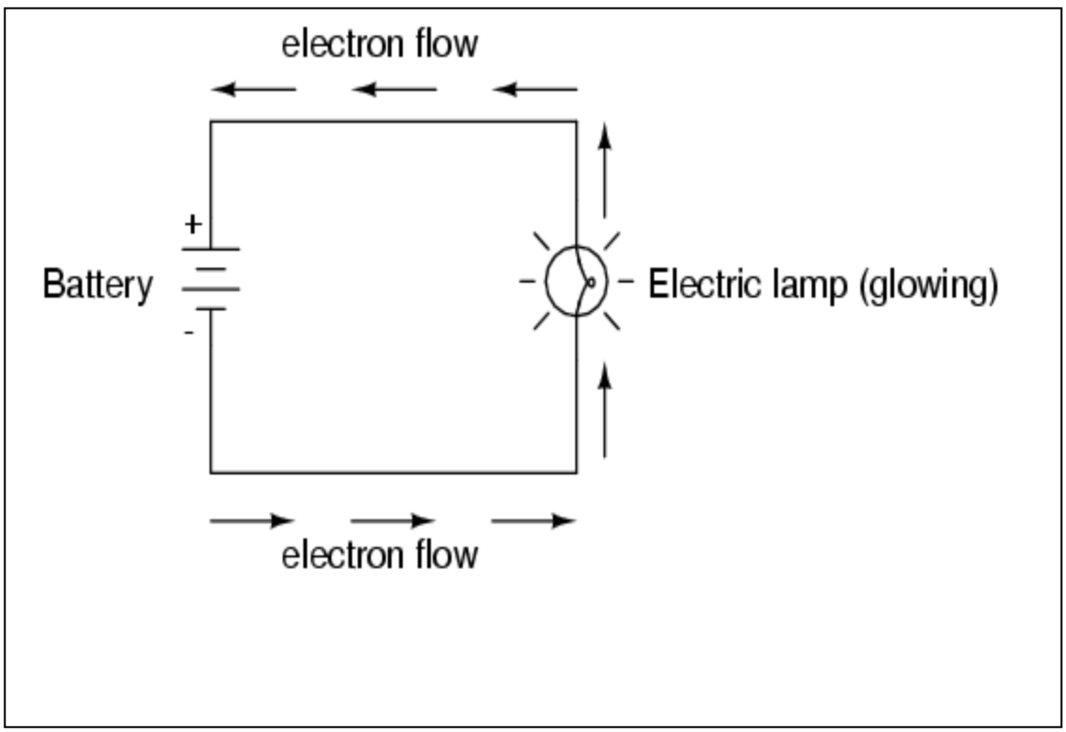
- Integrated circuit (also called a chip) A piece of silicon on which multiple gates have been embedded

Terms

Concepts:

- Voltage (V), measured in volts (V)
- Current (I), measured in amperes (A)
- Resistance (R), measured in ohms (Ω)





Given values of voltage (V) and resistance (R) , calculate the amount of current (I) in a circuit,

Circuit

- Ohm's law
  - V=I*R in volts
  - I=V/R in amphs
  - R=V/I in ohms
- ALWAYS mark the direction of current in a circuit

Example 1 : Assume that voltage V = 12 V and Resistance (R) 3 amps , Find the Current (I) ?

Current (I) = Voltage (V) / Resistance (R) = 12 / 3 = 4 ohms.

Example 2 : Calculate the amount of Resistance (R) in a circuit, given values of Voltage (V) and Current (I) ?

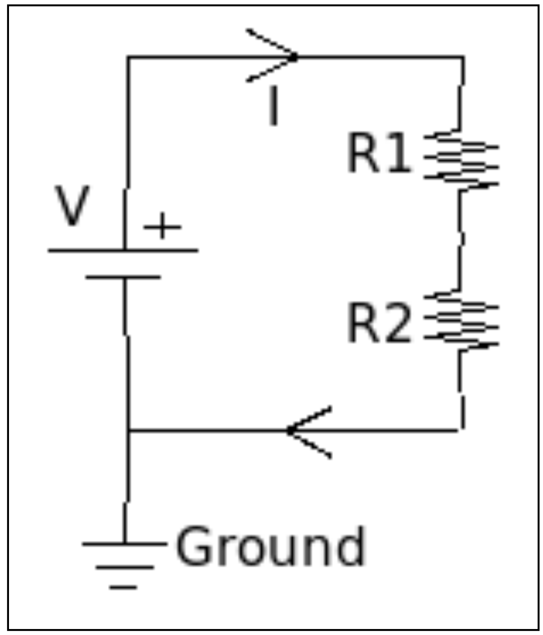
Assume that V = 36 and I = 4 ;
Using Ohms Law R = V / I 36 / 4 = 9 Ohms.

Example 3 : calculate the amount of Voltage supplied by a battery, given values of Current (I) and Resistance (R):

Assume that R = 7 ohms and I = 2 amphs ;

Then V = I * R = 2 * 7 = 14 volts

Series Circuit

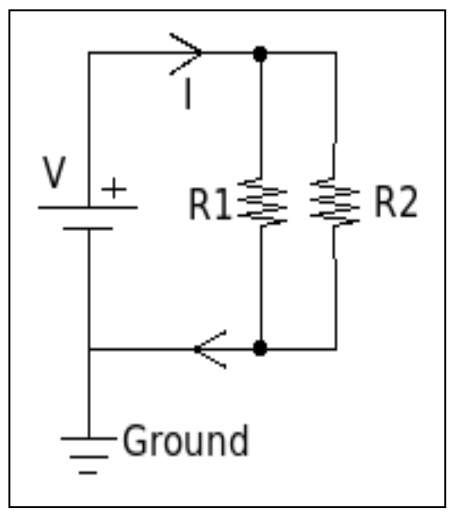


- V=V1+V2

  = I*R1 + I*R2

- The two components divides the voltage.
- The current through all of the components is equal.
- The voltage of the source is the sum of the voltages across each of the components.

Parallel Circuit



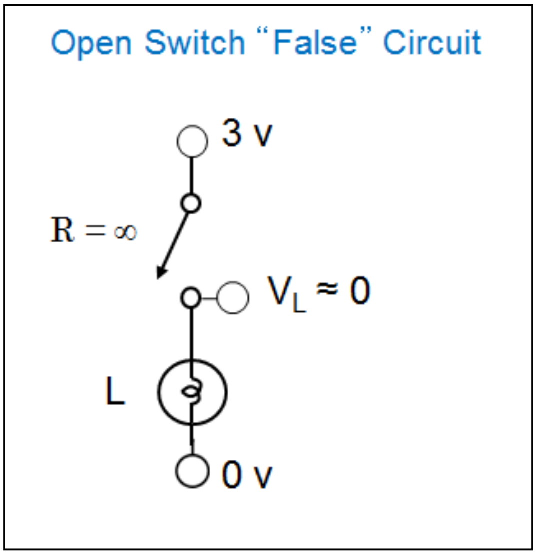
- I = I1 + I2

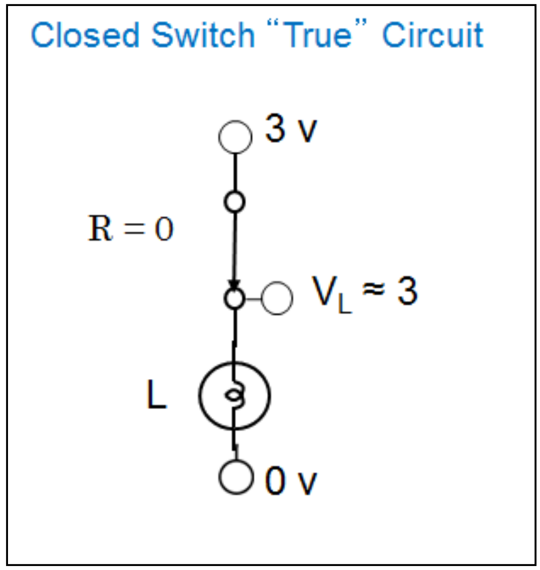
  = V/R1 + V/R2

- The two components divides the current.
- The voltage across each of the components is equal.
- The total current is the sum of the currents through the components

Device:

- Switch: Device whose resistance can be either 0 or ∞.
  - When 0, it's said to be Closed" …   "On"
  - When ∞, it's said to be "Open" …   "Off"





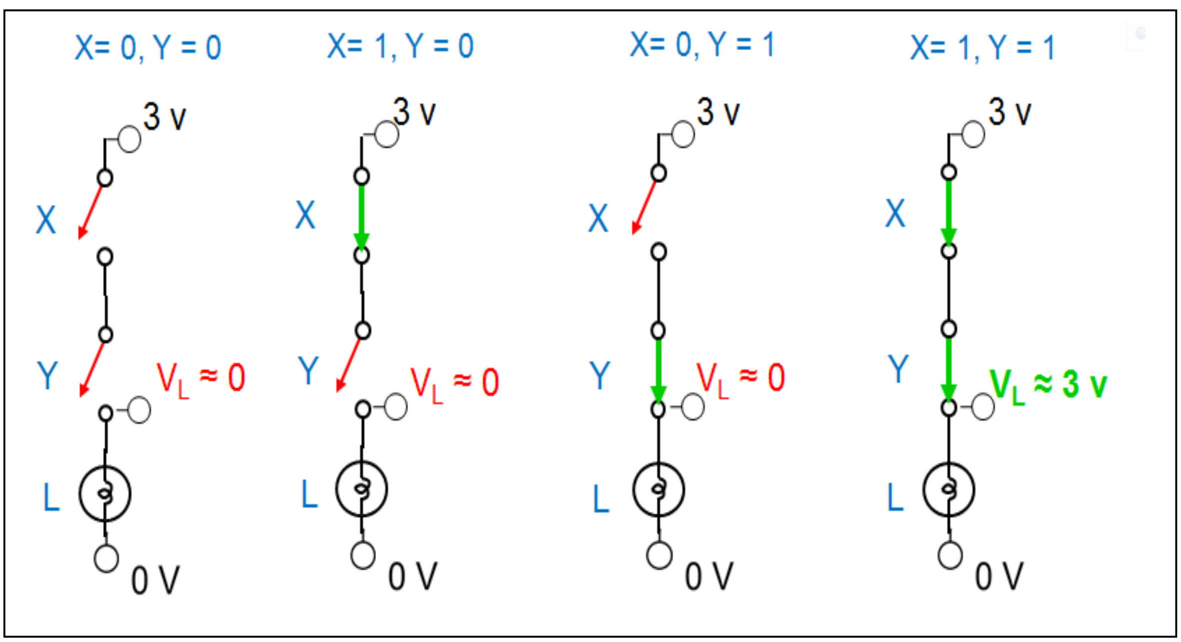
More Concepts

- In digital circuits (almost always):
  - High voltage represents TRUE or 1
  - Low voltage represents FALSE or 0

X	Y	L
0	0	0
1	0	0
0	1	0
1	1	1

- What happens when we use switches together?

The logical AND gate



CMOS Transistor Switches

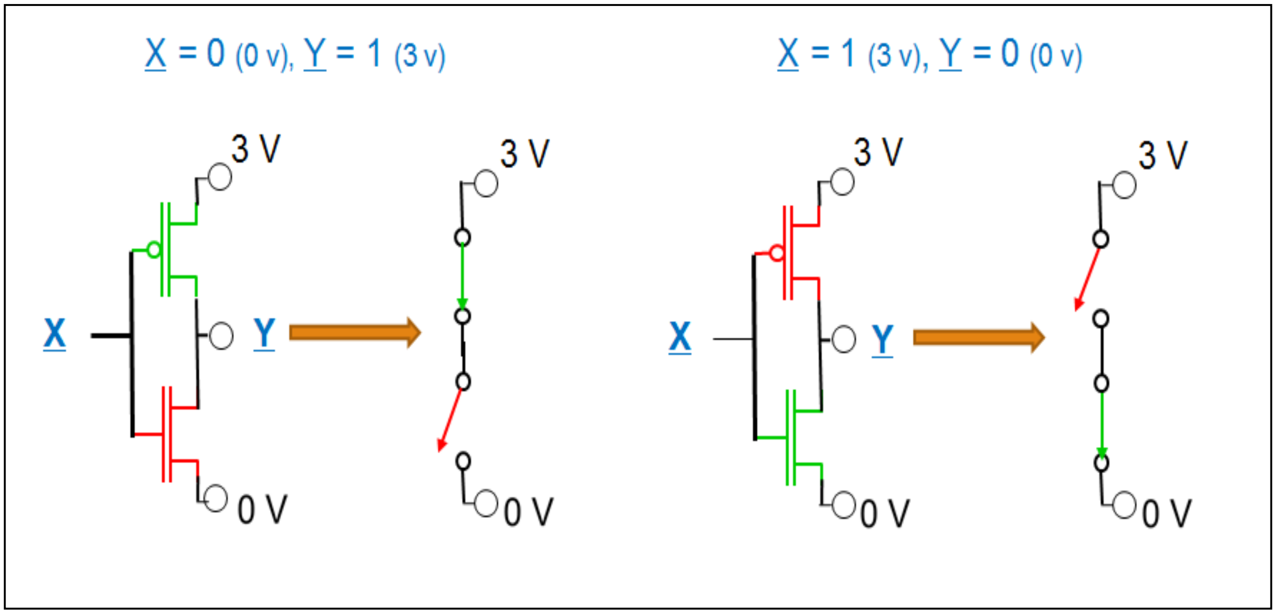
Switches that open and close when logical 1's and 0's are applied to their gates.

N-channel  3 V turns it on, 0 V turns it off

P-channel  0 V turns it on, 3 V turns it off.

Logical Invertor ~~ NOT Gate

X	Y
0	1
1	0



Reading Assignment

- Chapter 2: Sections 1 and 2